End-to-End Learning of User Equilibrium with Implicit Neural Networks

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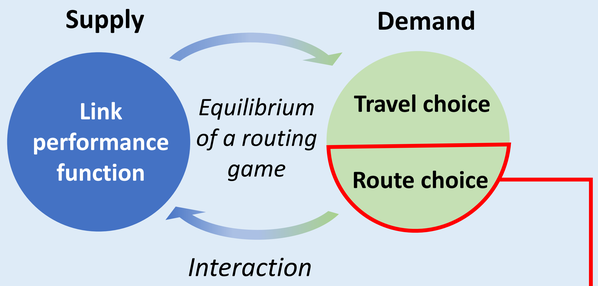
1 Classic Transportation Network Modeling Diagram



The selection of behavior model is **based on modelers' belief** rather than being the outcome of a calibration process against empirical flow data.

Research problem

We aim to transform the network equilibrium modeling paradigm via an **"end-to-end" framework** that **directly learns travel choice preferences and the equilibrium state from empirical data.**

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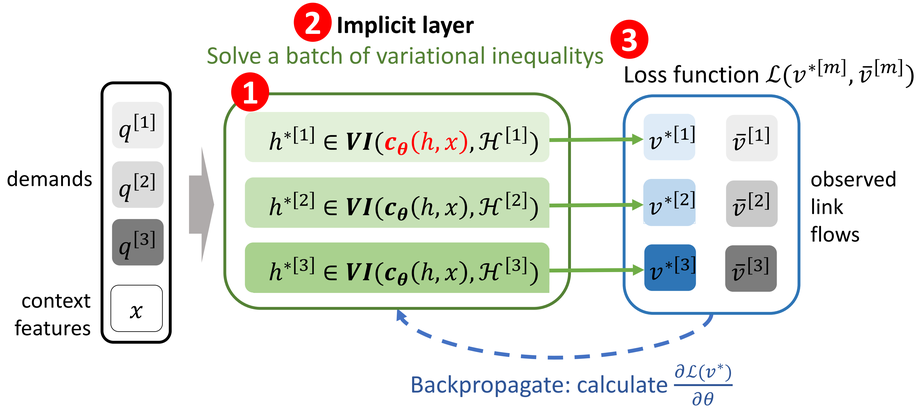
Pre-print available at SSRN!

2 Proposed "End-to-end" Learning Framework

Example: *Consider context features x, origin-destination demands q and observed link flows $\bar{v}$ are available for three months, where $q^{[1]}$ represents the demand of the first month.*

1 **Approximate** travelers' cost function $c_\theta(h, x)$ with neural networks, where h is path flow and θ is neural network parameters.



2 **Encapsulate** the user equilibrium conditions with variational inequalities or a implicit layer, i.e.,

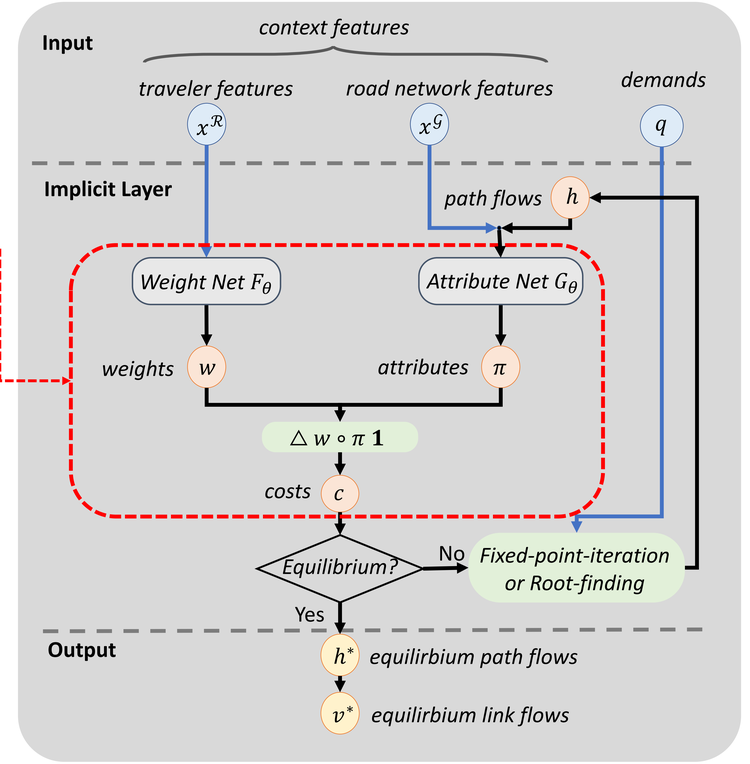
$$c_\theta\left(h^{[m]}, x\right)^T \left(h^{[m]*} - h^{[m]}\right) \geq 0, \forall h \in \mathcal{H}^{[m]}$$

where $\mathcal{H}^{[m]}$ is the feasible region of path flows.

3 **Train** the neural networks by minimizing the differences between the computed equilibrium flows v^* and flow observations $\bar{v}$.

Learning challenges

- How to design neural networks so that equilibrium exists?
- How to simultaneously solve a batch of variational inequalities?
- How to differentiate through the equilibrium solutions?



3 Case study

Sioux Falls Network. Consider a non-link-additive cost function.

Table 1. Mean Absolute Percentage Error of link flow predictions

Model	*Trained and tested on the same road network*	*Trained and tested on different road networks*
UE	15.5	14.2
SUE-logit	12.7	15.7
End-to-end	6.0	6.4

Inaccurately assuming a logit-based Stochastic User Equilibrium behavior model can cause bias in parameter estimation and mislead the flow prediction.