MICHIGAN ENGINEERING CCAT

TRBAM-24-04054

End-to-End Learning of User Equilibrium: Expressivity, Generalization, and Optimization

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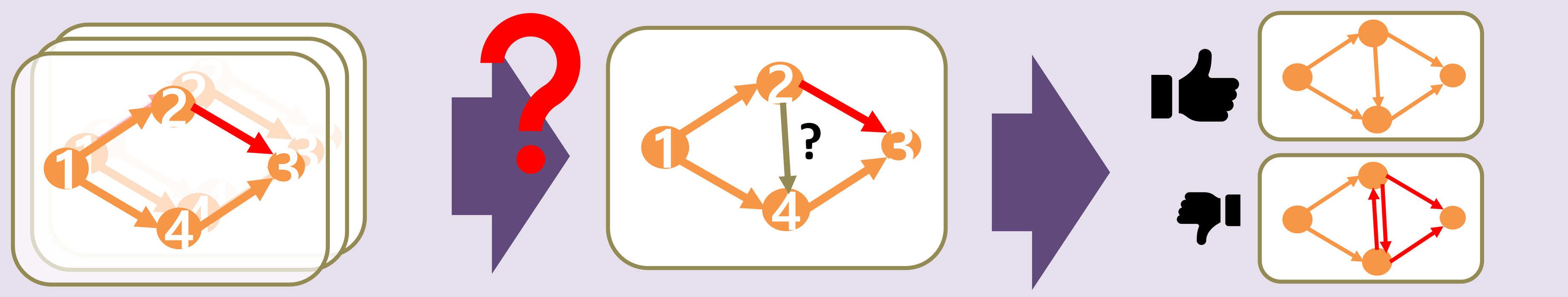
1 Research Question

Can neural networks help planners learn a “better” traffic assignment model from real-world data ?



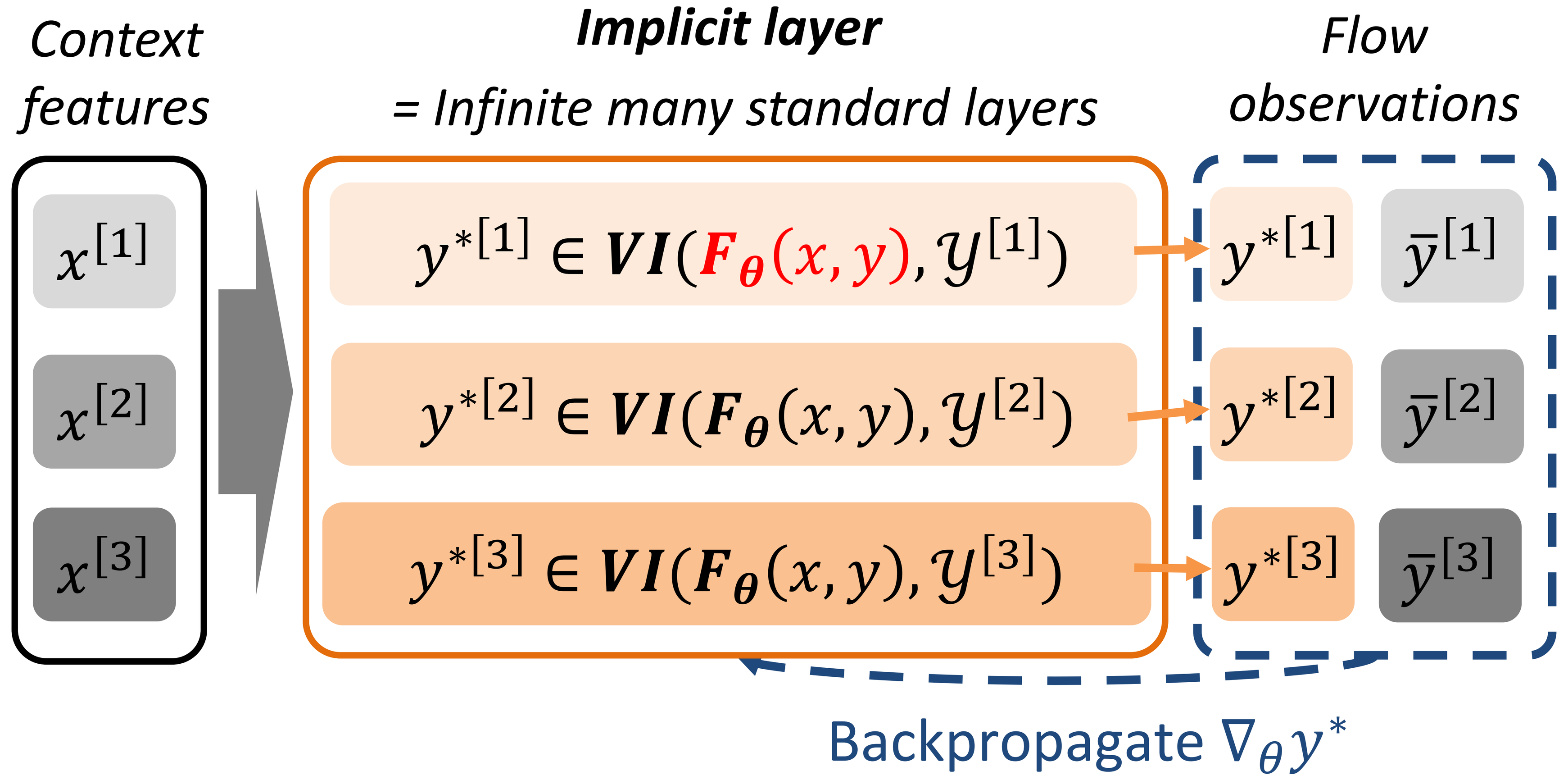
6 Main Takeaway

1. Utilizing neural networks' representation power, our end-to-end framework **learns s a range of "well-defined" user equilibria** from observed traffic states, maintaining controlled errors.
2. Auto-differentiation boosts scalability and ensures local convergence in training.
3. Using single-step network loading instead of user equilibrium during training may compromise performance.

2 End-to-End Framework

Consider planners observe weather and flows for three days.

a **Approximate** the travel cost function $F_\theta(x,y)$ with neural networks.

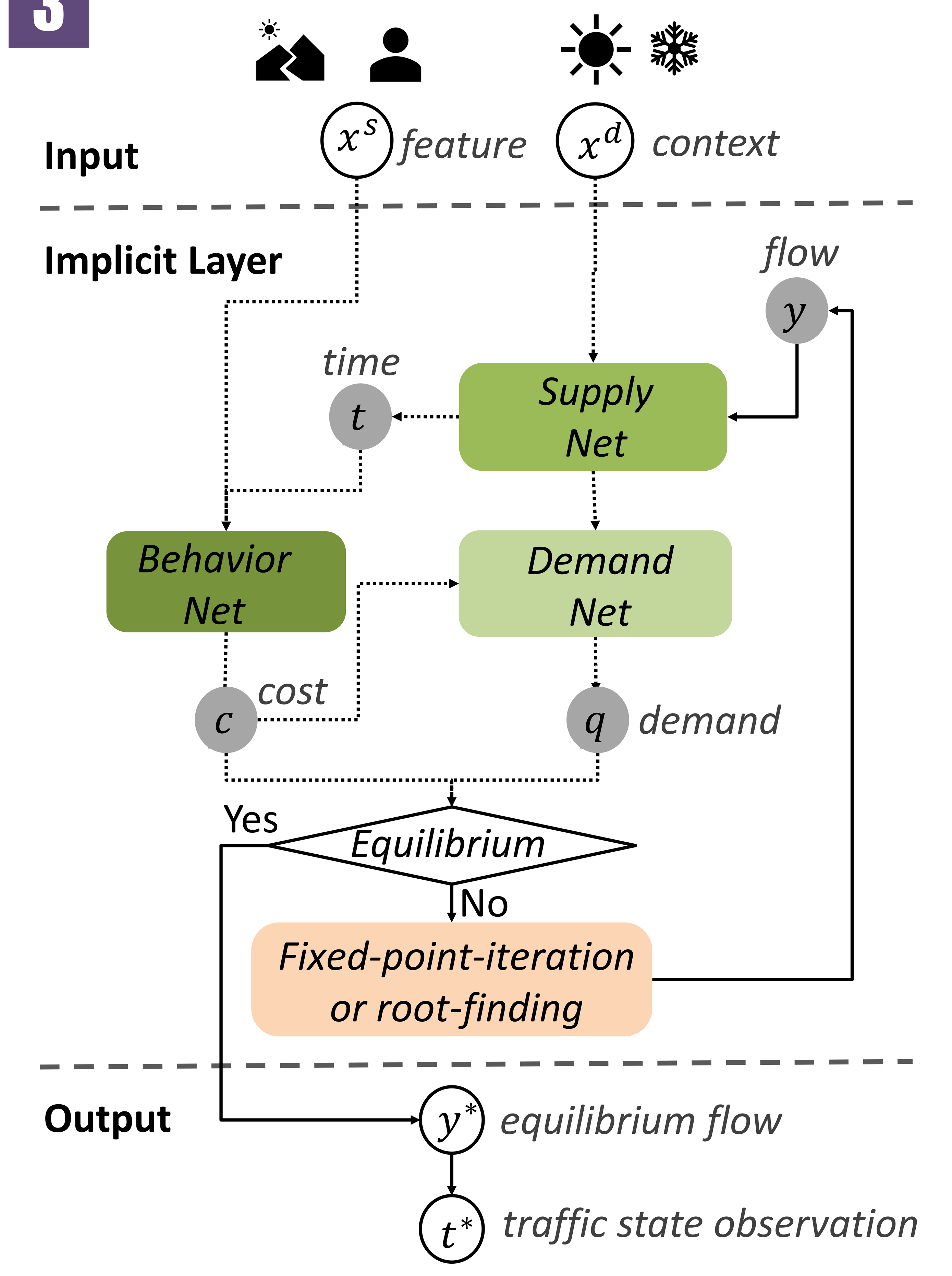


b **Encapsulate** user equilibrium with an implicit layer and solve a batch of variational inequalities.

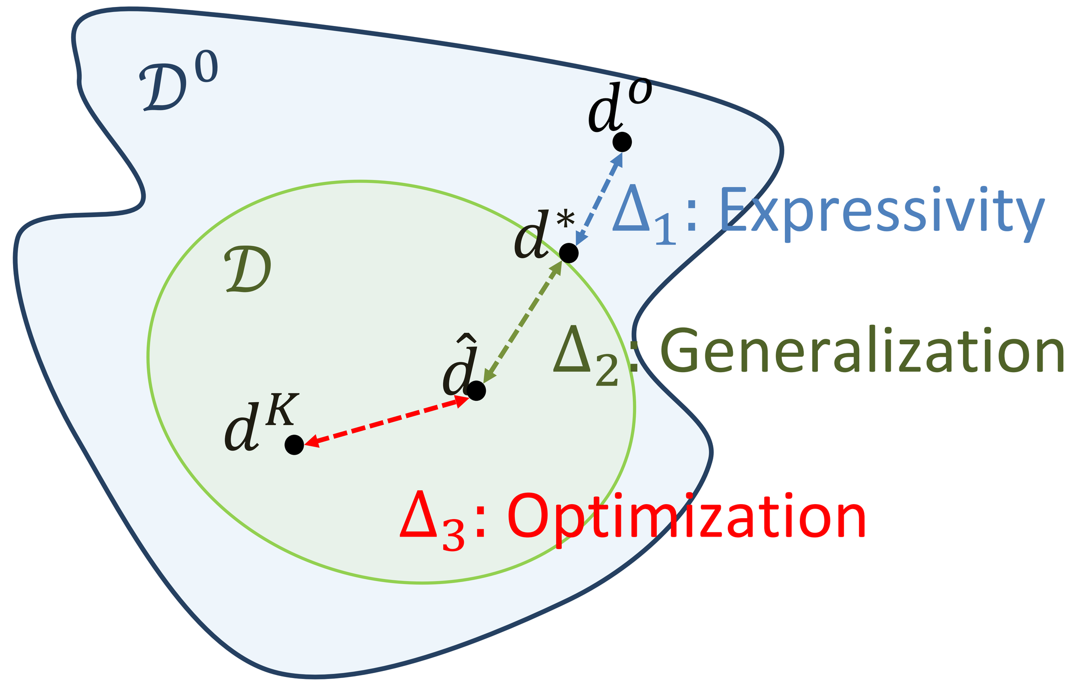
$$\langle F_\theta(x,y^*),\ y - y^* \rangle \geqslant 0, \quad \forall y \in \mathcal{Y}$$

c **Auto-differentiate** through the equilibrium states to learn parameter θ to minimize fitting error/

3



4 Framework analysis



Total error $\Delta \leq$	Δ_1	Δ_2	Δ_3
# Parameters ↑	↓	↑	=
# Samples ↑	=	↓	↑
# Equilibrium approximation ↑	=	=	↓

5 Case Study

Chigago sketch

Model	Inverse demand function	Link performance function	Number of Parameter	Link time prediction error (%)
Functional	Context-dependent exponential function	Context-dependent BPR	4	2.93
Linear	Linear function	Standard BPR	6	12.6
End-to-end	Residual neural network	Physics-informed neural network	232	9.5

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